4) $\dfrac{Ln(1+3tgx)}{x^3}=\dfrac{1}{x^2}.\dfrac{Ln(1+3tgx)}{x}$

Soit $f(x)=Ln(1+3tgx)$

$f'(x)=\dfrac{3(1+tg^2x)}{1+3tgx}$

$f'(0)=3\ ,\ f(0)=0$

$\lim\limits_{x\to0}\dfrac{f(x)}{x}=f'(0)=3$

$\Rightarrow\lim\limits_{x\to0}\dfrac{Ln(1+3tgx)}{x}=3$

$\lim\limits_{x\to0}\underbrace{\dfrac{1}{x^2}}_{+\infty}.\underbrace{\dfrac{Ln(1+3tgx)}{x}}_{3}=+\infty$

5) $f(x)=\dfrac{(x-1)(x+4)}{Lnx}$

$f(x)=\dfrac{x+4}{\left(\dfrac{Lnx}{x-1}\right)}$

$\lim\limits_{x\to1}f(x)=\lim\limits_{x\to1}\underbrace{\dfrac{x+4}{\left(\dfrac{Lnx}{x-1}\right)}}_{1}=5$

6) $\lim\limits_{x\to0^+}\sqrt{x}.Lnx\overset{x=\sqrt{x}}{=}\lim\limits_{X\to0^+}X.Ln(X^2)$

$=\lim\limits_{X\to0^+}2XLnX=0$

7) $\lim\limits_{x\to0^+}\sqrt{x}.(Lnx)^{10}\qquad X=\sqrt{x}$

$=\lim\limits_{X\to0^+}X\left[Ln(X^2)\right]^{10}$

$=\lim\limits_{X\to0^+}2^{10}.(XLn^{10}X)=0$

8) $\lim\limits_{x\to0}\dfrac{Ln(1+3x)}{\sin x}$

$=\lim\limits_{x\to0}\dfrac{\overbrace{Ln(1+3x)}^{3}}{\underbrace{\dfrac{\sin x}{x}}_{1}}=3$

EX 5 :

1) $f(x)=\dfrac{1}{x^4}+\dfrac{1}{x^3}+\dfrac{1}{x^2}+\dfrac{1}{x}$

$I=IR_+^*$

$F(x)=-\dfrac{1}{3x^3}-\dfrac{1}{2x^2}-\dfrac{1}{x}+Ln|x|$

$F(x)=Lnx-\dfrac{2+3x+6x^2}{6x^3}$

2) $f(x)=\dfrac{3x-4}{x+5}\ ;\ I=]-5,+\infty[$

$f(x)=\dfrac{3(x+5)-19}{x+5}$

$f(x)=3-\dfrac{19}{x+5}$

$F(x)=3x-19.Ln|x+5|$

$F(x)=3x-19Ln(x+5)$

3) $f(x)=\dfrac{\sin x-\cos x}{\cos x+\sin x}$

$f(x)=\dfrac{-(\cos x+\sin x)'}{\cos x+\sin x}$

$F(x)=-Ln|\cos x+\sin x|$

$F(x)=-Ln(\cos x+\sin x)$

$car\ \ x\in\left]0,\dfrac{\pi}{2}\right[$

4) $f(x)=\dfrac{2x+1}{x^2+x}$

$F(x)=Ln|x^2+x|$

$F(x)=Ln(x^2+x)\ \ ;x\in\ ]0,+\infty[$

5) $f(x)=\dfrac{\dfrac{1}{x}}{Lnx}$

$F(x)=Ln|Lnx|$

$F(x)=Ln(Lnx)\ \ pour\ x\in\ ]1,+\infty[$

6) $f(x)=\dfrac{1}{x}(Lnx)^3$

$F(x)=\dfrac{1}{4}(Lnx)^4$

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